Non-uniform thermal magnetization noise in thin films: application to GMR heads

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A general scheme is developed to analyze the effect of non-uniform thermal magnetization fluctuations in a thin film. The normal mode formalism is utilized to calculate random magnetization fluctuations. The magnetization noise is proportional to the temperature and inversely proportional to the film volume. The total noise power is the sum of normal mode spectral noises and mainly determined by spin-wave standing modes with an odd number of oscillations. The effect rapidly decreases with increasing mode number. An exact analytical calcutaion is presented for a two-cell model.

I. INTRODUCTION

Recently thermal magnetization fluctuations have been shown to be an important issue for thin film magnetoresistive heads used for high-density magnetic recording [1]. Noise estimates have been initially made for the simplest model of a uniformly magnetized film [1-3]. In order to develop a more realistic model it is necessary to analyze the role of non-uniform magnetization fluctuations. In this paper we propose a general scheme to calculate the effect of non-uniform thermal magnetization fluctuations in an ultrathin film. After discretizing the film into magnetic cells, the well-known normal mode formalism is utilized. By doing so we represent the magnetization noise in terms of independent standing spin waves subjected to white thermal noise. As an example, the explicit solution is given for a two-mode model.

II. CALCULATION SCHEME

Let us consider a thin film of the volume $V = d_x \times d_y \times d_z$, where $d_x \ll d_y$, d_z are the thickness, width and length, respectively. The thickness is assumed to be of the order or less than exchange length ($d_x \leq \sqrt{A}/M_s$) and therefore non-uniform magnetization dynamics along the x direction will be neglected. The film is discretized in the yz plane into N identical magnetic cells. Each cell is assumed to be uniformly magnetized and can be represented as a classical spin $\mathbf{S}_j = -\mathbf{M}(\mathbf{r}_j)V_c/\hbar\gamma$, where $\mathbf{M}(\mathbf{r}_j)$ is the magnetization in the cell ($|\mathbf{M}(\mathbf{r}_j)| = M_s$), $V_c = V/N$ is the cell volume, M_s is the saturation magnetization and γ is the gyromagnetic ratio. Planck's constant $\hbar$ is used for convenience as a dimensional variable.

Our approach is purely classical and final result does not contain $\hbar$.

The energy of the spin system can be written as

$$\mathcal{E} = \mathcal{E}_{\text{anis}} + \mathcal{E}_Z + \mathcal{E}_{\text{dd}} + \mathcal{E}_{\text{exch}}, \tag{1}$$

where

$$\mathcal{E}_{\text{anis}} = \frac{\hbar\gamma}{2S} \sum_{j=1}^{N} \left[-H_{\text{K},j}^{(\text{in})} (\mathbf{n}_j \cdot \mathbf{S}_j)^2 + H_{\text{K},j}^{(\perp)} (S_j^x)^2 \right] \tag{2}$$

is the energy of anisotropy, $\mathbf{n}_j$ is the unit vector along the axis of in-plane uniaxial anisotropy, $H_{\text{K},j}^{(\text{in})}$ is the local uniaxial anisotropy field, $H_{\text{K},j}^{(\perp)}$ is the local hard-axis anisotropy field.

$$\mathcal{E}_Z = \hbar\gamma \sum_{j=1}^{N} \mathbf{H}_j \cdot \mathbf{S}_j \tag{3}$$

is the Zeeman energy with local magnetic fields $\mathbf{H}_j$. $\mathcal{E}_{\text{dd}}$ is the energy of magnetostatic interactions between cells and $\mathcal{E}_{\text{exch}}$ describes the exchange interaction between the nearest neighboring cells. Boundary conditions can be taken into account by choosing corresponding anisotropy fields.

The calculation procedure consists of the following steps. From (1) it is necessary to find the equilibrium direction in the yz plane for each spin (cell). This direction is defined by the angle θ_j relative to z axis. Then we represent the spin in its local "quantization axes" $\mathbf{x}_j$, $\mathbf{y}_j$, $\mathbf{z}_j$, which are associated with the spin equilibrium direction ($\mathbf{S}_j \parallel \mathbf{z}_j$):

$$S_j^y = S_j^{y_j} \cos\theta_j + S_j^{z_j} \sin\theta_j, \tag{4}$$
$$S_j^z = S_j^{z_j} \cos\theta_j - S_j^{y_j} \sin\theta_j, \quad S_j^x = S_j^{x_j}.$$

Now we analyze small deviations for each spin from equilibrium. It is convenient to describe small magnetization oscillations by conventional spin-wave technique. Using a linearized Holstein-Primakoff transformation [4], we introduce complex variables a^*, a, which are classical analogs of creation and annihilation operators:

$$S_j^{z_j} = -S + a_j^* a_j, \quad S_j^{y_j} \simeq (a_j + a_j^*)\sqrt{2S}/2, \tag{5}$$
$$S_j^{x_j} \simeq (a_j - a_j^*)\sqrt{2S}/2i.$$

The quadratic part of the energy (1) can be written in the form:

$$\mathcal{E}^{(2)}/\hbar = \sum_{k,l} \left(A_{kl} a_k^* a_l + \frac{B_{kl}}{2} a_k^* a_l^* + \frac{B_{kl}^*}{2} a_k a_l \right), \quad (6)$$

where $A_{kl} = A_{lk}^*$ and $B_{kl} = B_{lk}$.

To eliminate the nondiagonal terms from (6) we shall use the linear canonical transformation [5]:

$$a_j = \sum_{k=1}^{N} \left(u_{jk} c_k + v_{jk}^* c_k^* \right), \quad (7)$$

where c_k and c_k^* describe complex variables of normal spin-wave modes. From gyromagnetic equations we have the algebraic equations for eigenvalues ω_k (standing spin wave frequencies) and eigenvectors u_{lk}, v_{lk}:

$$\omega_k u_{jk} = \sum_{l=1}^{N} (A_{jl} u_{lk} + B_{jl} v_{lk}),$$

$$-\omega_k v_{jk} = \sum_{l=1}^{N} (A_{jl}^* v_{lk} + B_{jl}^* u_{lk}). \quad (8)$$

This system must be supplemented by the orthogonality and normalization conditions:

$$\sum_{j=1}^{N}(u_{jl} u_{jk}^* - v_{jl} v_{jk}^*) = \delta_{lk}, \quad \sum_{j=1}^{N}(u_{jl} v_{jk} - u_{jk} v_{jl}) = 0,$$

$$\sum_{k=1}^{N}(u_{jk} u_{lk}^* - v_{jk} v_{lk}^*) = \delta_{jl}, \quad \sum_{k=1}^{N}(u_{jk} v_{lk} - u_{lk} v_{jk}) = 0. \quad (9)$$

The resulting quadratic energy form describes N independent normal modes (harmonic oscillators):

$$\mathcal{E}^{(2)}/\hbar = \sum_{k=1}^{N} \omega_k c_k^* c_k. \quad (10)$$

The evolution of each normal spin-wave mode is characterized by frequency (ω_k) and relaxation rate (η_k): $c_k \propto \exp(-i\omega_k t - \eta_k t)$. From equipartition, white thermal noise ($k_B T$) is applied to each independent mode. Using the solution for a damped harmonic oscillator with additive noise (see, [2], [3]), we calculate the spectral density of the magnetic fluctuations:

$$S_{c_k^* c_l}(\omega) = \int_{-\infty}^{\infty} \langle c_k^*(t) c_l(0) \rangle e^{-i\omega t} dt$$

$$= \delta_{kl} \frac{2(\eta_k/\omega_k)}{(\omega_k - \omega)^2 + \eta_k^2} \frac{k_B T}{\hbar}, \quad (11)$$

where T is the temperature, $\langle ... \rangle$ means thermal averaging. Note that the total system has thermal energy $N k_B T$.

III. APPLICATION TO A MR HEAD

As a general application to a GMR head we take the z direction to be the cross track direction along which the current is applied and thus the deviations of magnetization in the y direction are responsible for magnetoresistance. The power spectral density due to random magnetization fluctuations can be written as [1], [2]:

$$S_{VV}(\omega) = C_V^2 S_{m_y m_y}(\omega), \quad (12)$$

$$C_V \equiv I_{\text{bias}}(\partial R/\partial H_{\text{ext}})(\partial H_{\text{ext}}/\partial m_y), \quad (13)$$

where

$$S_{m_x m_x}(\omega) = \int_{-\infty}^{\infty} \langle \delta m_y(t) \delta m_y(0) \rangle e^{i\omega t} dt. \quad (14)$$

defines the magnetization noise. Here δm_y denotes the net random deviation of the normalized total magnetic moment:

$$\delta m_y = -\sum_{j}^{N} \delta S_j^{y_j} \cos \theta_j / NS. \quad (15)$$

Using Eqs. (5) and (7), we rewrite $\delta m_y(t)$ in terms of the normal mode variables $c_k(t)$ and $c_l^*(t)$. Thus, the correlation function (14) can be rewritten in terms of correlation functions $\langle c_k(t) c_l^*(0) \rangle$ and $\langle c_k^*(t) c_l(0) \rangle$. Utilizing (11), we finally obtain:

$$S_{m_y m_y}(\omega) = \frac{\gamma k_B T}{M_s V} \sum_{k=1}^{N} |W_k|^2 \frac{\eta_k}{\omega_k} F_k(\omega), \quad (16)$$

$$F_k(\omega) = \frac{1}{(\omega_k - \omega)^2 + \eta_k^2} + \frac{1}{(\omega_k + \omega)^2 + \eta_k^2},$$

$$W_k = N^{-1/2} \sum_{j=1}^{N} (u_{jk} + v_{jk}) \cos \theta_j. \quad (17)$$

Thus the magnetization noise power (16) is proportional to the temperature and inversely proportional to the film volume, and is the sum of mode spectral functions $(\eta_k/\omega_k) F_k(\omega)$ with some "weights" W_k (17). In the case of $N = 1$ we obtain the result of [2], [3].

The summation of amplitudes of spin waves $u_{jk} + v_{jk}$ in (17) qualitatively means an integration of oscillations in the standing mode (see, Fig.1). For example, if equilibrium is along the z direction ($\theta_j = 0$) the integration over the modes in Fig.1b and 1d vanish and only modes from Fig.1a and 1c will contribute. Due to partial cancellation in the odd spin-wave standing mode (see, e.g., Fig.1c) the weight rapidly decreases with increasing mode wave number.

IV. EXAMPLE: TWO MODE MODEL

Let us consider now the case of two-cell discretization (Fig.2). We take into account the "easy" anisotropy field H_K along the z axis and the "hard" axis field $4\pi M_s$ perpendicular to the plane. In this example only magnetostatic interaction between cells is included taken in the form: $(\hbar\gamma)^2(\zeta\mathbf{S}_1 \cdot \mathbf{S}_2 - \zeta_y S_1^y S_2^y - \zeta_z S_1^z S_2^z)/V_c^3$, where ζ, ζ_y and ζ_z are the factors, which depend on the magnetostatic energy for a given film geometry. For example, for a film of $4 \times 180 \times 250$ nm^3 $\zeta \approx 0.23$, $\zeta_y \approx 0.17$ and $\zeta_z \approx 0.063$.

The energy quadratic form in terms of spin deviations (5) can be written as:

$$\mathcal{E}^{(2)}/\hbar = A(a_1^* a_1 + a_2^* a_2) + B(a_1^* a_2 + a_2^* a_1) + C(a_1 a_2 + a_1^* a_2^*) + (D/2)(a_1 a_1 + a_1^* a_1^* + a_2 a_2 + a_2^* a_2^*), \quad (18)$$

where

$$A = \gamma\{H_K(\cos^2\theta - \frac{1}{2}\sin^2\theta) - M_s[\zeta - 3(\zeta_y \sin^2\theta + \zeta_z \cos^2\theta)] + 2\pi M_s + H_0\cos(\theta_H - \theta)\},$$
$$B = \gamma M_s\zeta + C, \quad D = -\gamma(2\pi M_s + H_K\sin^2\theta/2),$$
$$C = -3\gamma M_s(\zeta_y\cos^2\theta + \zeta_z\sin^2\theta)/2.$$

From (8) and (9) we obtain:

$$u_{11} = u_{21} = uU_1 + vV_1, \quad v_{11} = v_{21} = uV_1 + vU_1, \quad (19)$$
$$u_{12} = -u_{22} = uU_2 + vV_2, \quad v_{12} = -v_{22} = uV_2 + vU_2,$$

where

$$U_k = \sqrt{\frac{\Omega - (-1)^k B + \omega_k}{4\omega_k}}, \quad v = -\frac{D}{|D|}\sqrt{\frac{A - \Omega}{2\Omega}},$$

$$V_k = (-1)^k\sqrt{\frac{\Omega - (-1)^k B - \omega_k}{4\omega_k}}, \quad u = \sqrt{\frac{A + \Omega}{2\Omega}},$$

$$\omega_k = \sqrt{[\Omega - (-1)^k B]^2 - C^2}, \quad \Omega = \sqrt{A^2 - |D|^2}. \quad (20)$$

For equilibrium in the cross track direction the two modes correspond to coherent and fanning about the z direction (similar to the modes illustrated in Fig.1a and 1b, respectively). From Eq.(19) we can see that the "weight" of the second mode is always equal to zero: $W_2 \propto (u_{12} + v_{12} + u_{22} + v_{22})\cos\theta = 0$. Thus, the two-mode model is reduced with some modification to the case of the single-mode model.

[1] N. Smith and P. Arnett, Appl. Phys. Lett. **78**, 1448 (2001).
[2] V. L. Safonov and H. N. Bertram, submitted to J. Appl. Phys. (2001); arXiv: cond-mat/0103624.
[3] H. N. Bertram, Z. Jin, and V. L. Safonov, TMRC 2001, to be published in IEEE Trans. Mag. (2002).
[4] T. Holstein and H. Primakoff: Phys. Rev. **58**, 1098 (1940).
[5] S. V. Tyablikov, *Methods in the quantum theory of magnetism* (Plenum, NY, 1967) p.105.

Figure captions.
Fig. 1.
Standing spin-wave modes (qualitatively) with odd (a, c) and even (b, d) numbers of oscillation humps.
Fig.2
Film with two-cell discretization.